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EDUCATION

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NATIONAL SENIOR CERTIFICATE *NASIONALE SENIORSERTIFIKAAT*

GRADE/*GRAAD* 12

SEPTEMBER 2023

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: **150**

This marking guideline consists of 14 pages.
Hierdie nasienriglyn bestaan uit 14 bladsye.

QUESTION 1/*VRAAG 1*

21,86 30,14

19 22 24 27 29 30 31 Put in order !! NB !!!

Data Set / *Datastel*: 29 27 24 31 22 19 30			
1.1.1(a)	Mean / *Gemiddelde* $= \frac{182}{7} = 26$ ✓ (2)	✓✓ mean / *gemiddelde*	(2)
1.1.1(b)	Standard deviation / *standaardafwyking* = 4,14 ✓ (1)	✓ SD / *standaardafwyking*	(1)
1.1.2	{26 – 4,14; 26 + 4,14} = {21, 86; 30, 14} ✓ ✓ ✓ (3) 5 players / *spelers* ✓	✓✓ calculations / *berekeninge* ✓ answer / *antwoord*	(3)
1.1.3	Rugby team has the same average number of push-ups. The rugby team results were clustered around the mean because of the smaller standard deviation. less variation *Rugbyspan het dieselfde gemiddelde aantal opstote. Die rugbyspan se uitslae was rondom die gemiddelde gegroepeer agv die kleiner standaardafwyking.* ✓ ✓ (2)	✓✓ for any two valid comments using SD and the mean *Vir enige twee geldige opmerkings in gebruik van standaardafwyking en die gemiddelde*	(2)
1.2.1	50% ✓ M = 30 (1)	✓ answer / *antwoord*	(1)
1.2.2 P10	Mean / *Gemiddelde* ✓ $\bar{x} - M > 0$ ✓ Distribution skewed to the right. (positively skewed) (2) *Verspreiding is skeef na regs (positief skeef)*	✓ answer / *antwoord* ✓ reason / *rede*	(2)
			[11]

QUESTION 2/*VRAAG 2*

Third Term % *Derde Kw. %*	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final Term % *Finale Kw. %*	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71

2.1	a = 9, 035 ✓ b = 0, 895 ✓ $\hat{y} = 9{,}035 + 0{,}895x$ ✓ (3) 3 dp	✓ for a / *vir a* ✓ for b / *vir b* ✓ for equation / *vir vergelyking*	(3)
2.2	r = 0, 98 ✓ 2 dp (1)	✓ answer / *antwoord*	(1)
2.3.1	$y = 9{,}035 + 0{,}895(48)$ ✓ $\in \mathbb{N}_0$ (2) $y \approx 52$ ✓	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
2.3.2	correlation is very strong **OR** 48 is within domain of regression line. (1) *korrelasie is baie sterk **OF** 48 is binne die gebied van die regressie-lyn.*	✓ answer / *antwoord*	(1)
2.4.1	50% is outside the domain of the line (data set) **OR** (50 ; 80) is an outlier. ✓ (1) *50% is buite die gebied van die lyn (datastel) **OF** (50 ; 80) is 'n uitskieter*	✓ answer / *antwoord* 54% + explanation	(1)
2.4.2	Increase the gradient / *Vermeerder die gradiënt* (1) ✓	✓ answer / *antwoord*	(1)
			[9]

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1.2.2 $Q_3 - M = 48 - 30 = 18$ $\quad M - Q_1 = 30 - 26 = 4$

$18 > 4$ $\therefore$ data skewed to the right

$\therefore \quad \bar{x} - M > 0$

$\therefore \quad \bar{x} > M$

2.4 1. T3 50%

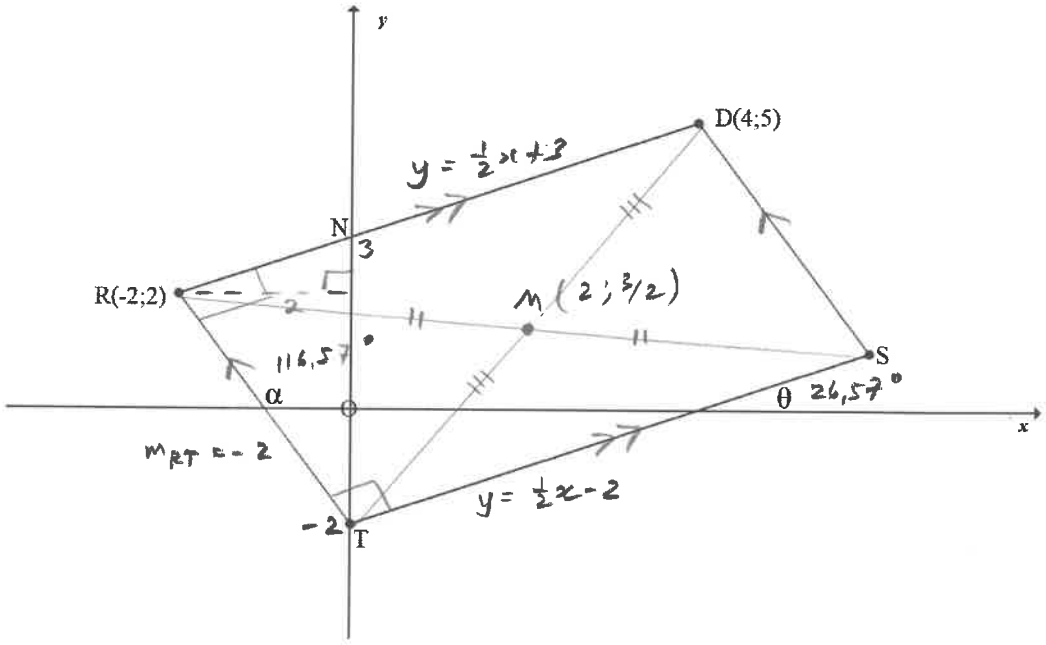
$\therefore$ Final : $y = 9{,}035 + 0{,}895(50)$

$= 53{,}785$

$\approx 54\%$

$\therefore$ 80% is unreliable as it is very different to 54%

QUESTION 3 / *VRAAG 3*



3.1	T(0; -2) ✓ yint (1)	✓ answer / *antwoord*	(1)
3.2.1	$m_{RT} = \frac{2-(-2)}{-2-0} = -\frac{4}{2} = -2$ ✓✓	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.2.2	$\tan\theta = \frac{1}{2}$ ✓ $\therefore \theta = 26{,}57°$ ✓ $\tan\alpha = -2$ ✓ $\therefore \alpha = 116{,}57°$ ✓ $\therefore R\hat{T}S = 116{,}57° - 26{,}57° = 90°$ ✓ (5) **OR / *OF*** $m_{TS} = \frac{1}{2}$ $m_{RT} = -2$ $m_{TS} \times m_{RT} = -1$ $R\hat{T}S = 90°$	✓ for / *vir* $\tan\theta = \frac{1}{2}$ ✓ for / *vir* $\theta = 26{,}57°$ ✓ for / *vir* $\tan\alpha = -2$ ✓ for / *vir* $\alpha = 116{,}57°$ ✓ $R\hat{T}S = 90°$ **OR / *OF*** ✓ $m_{TS} = \frac{1}{2}$ ✓ $m_{RT} = -2$ ✓✓ product of gradients *produk van gradiënte* ✓ $R\hat{T}S = 90°$	(5)
3.3	$y = \frac{1}{2}x + c$ ✓ $m_{\perp}$ $2 = \frac{1}{2}(-2) + c$ ✓ (3) $c = 3$ $\therefore y = \frac{1}{2}x + 3$ ✓	✓ gradient / *gradiënt* ✓ substitution of point R or D *vervanging van punt* R *of* D ✓ answer / *antwoord*	(3)

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3.4	RDST is a parallelogram (opposite sides parallel) RDST *is 'n parallelogram (teenoorst. sye ewewydig)* Midpoint of DT / *Middelpunt van* DT: $\left(\frac{4+0}{2};\frac{5-2}{2}\right)=\left(2;\frac{3}{2}\right)$ Midpoint of RS is the same as midpoint of DT (diagonals bisect each other.) *Middelpunt van* RS *is dieselfde as die middelpunt van* DT *(hoeklyne halveer mekaar)* Midpoint of RS / *Middelpunt van* RS: $\left(2;\frac{3}{2}\right)$ **OR / *OF*** $S(6;1)$ Midpoint of RS / *Middelpunt van* RS: $\left(\frac{6-2}{2};\frac{1+2}{2}\right)=\left(2;\frac{3}{2}\right)$	✓ substitution in the MP formula *vervanging in die* MP *formule* ✓ S/R ✓ answer / *antwoord* **OR / *OF*** ✓✓ $S(6;1)$ ✓ answer / *antwoord*	(3)
3.5	N(0; 3) RN $=\sqrt{2^2+1^2}=\sqrt{5}$ RT $=\sqrt{2^2+4^2}=\sqrt{20}$ Area /*Oppervlakte* $=\frac{1}{2}\times\sqrt{20}\times\sqrt{5}$ $=5$ square units / *eenhede*^{2} **OR / *OF*** TN = 5 units / *eenhede* Height / *Hoogte* = 2 units / *eenhede* Area /*Oppervlakte* $=\frac{1}{2}\times 5\times 2$ Area /*Oppervlakte* = 5 square units / eenhede^2	✓ coordinates of N *koördinate van* N ✓ for / *vir* RN ✓ for / *vir* RT ✓ for the answer / *vir die antwoord* **OR / *OF*** ✓ TN = 5 units / *eenhede* ✓ Height/*hoogte* = 2 units / *eenhede* ✓ sub. into formula *vervanging in formule* ✓ answer / *antwoord*	(4)
			[18]

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3.4 RDST is a ||gm both prs opp sides quad ||

M is midpt of RS ✓SR diags ||gm bisect and TD

$x_M = \frac{0+4}{2}$ ✓ $y_M = \frac{-2+5}{2}$ T(0;-2) D(4;5)

$= 2$ $= 3/2$

$\therefore$ M(2; 3/2) ✓ (3)

3.5 N(0;3)

$\therefore$ NT = 5

$RT = \sqrt{(2-(-2))^2+(-2-0)^2}$ T(0;-2) R(-2;2)

$= \sqrt{20}$

$R\hat{T}N + 90° = 116,57°$ ext $\angle$ Δ

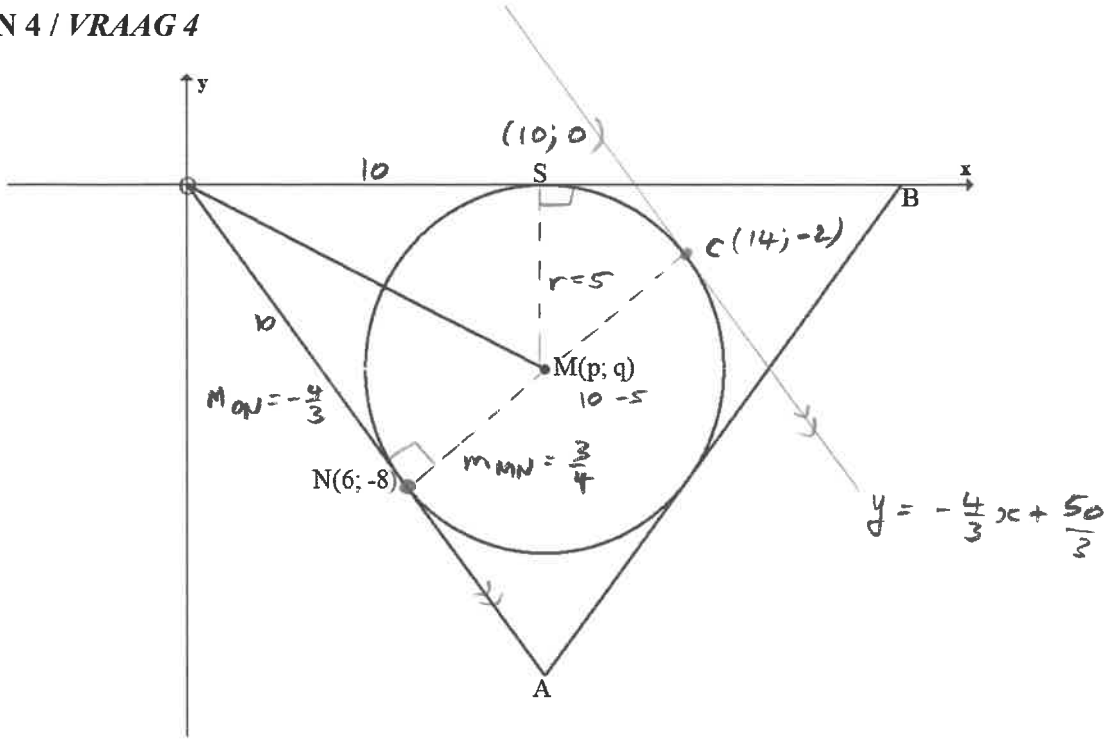
$R\hat{T}N = 26,57°$

area Δ RTN

$= \frac{1}{2}(5)(\sqrt{20})\sin 26,57°$

$= 5$

QUESTION 4 / *VRAAG 4*



4.1.1	$ON = \sqrt{(6-0)^2 + (-8)^2}$ ✓ $= \sqrt{100} = 10$ units / *eenhede* ✓	✓ substitution in correct formula *vervanging in korrekte formule* ✓ answer / *antwoord*	(2)
4.1.2	ON = OS (tangents from the same point) ✓SR *(raaklyne vanaf dieselfde punt)* ✓ $\therefore p = 10$ units / *eenhede*	✓ S and/*en* R ✓ answer / *antwoord* • tan's from ext common pt =	(2)
4.1.3	ON ⊥ NM (tan – radius) / *(raaklyn – radius)* ✓SR $m_{ON} = \frac{-8}{6} = \frac{-4}{3}$ ✓ $m_{NM} = \frac{3}{4}$ ✓	✓ S and/*en* R ✓ gradient of ON / *gradiënt van* ON ✓ gradient of NM *gradiënt van* NM	(3)
4.1.4	$m_{NM} = \frac{q+8}{10-6} = \frac{3}{4}$ ✓ $\quad \frac{q-(-8)}{10-6} = \frac{3}{4}$ $\frac{q+8}{4} = \frac{3}{4}$ $q = -5$ ✓ CA	✓ for subs and equating *vir vervanging en gelyk stel* ✓ answer / *antwoord*	(2)
4.2 CA 4.1.2	MS = r = 5 units / *eenhede* ✓ $(x-10)^2 + (y+5)^2 = 25$ ✓LRS ✓RTS	✓ radius / *radius* ✓ centre sub/ *vervang middelpunt* ✓ answer / *antwoord*	(3)
4.3	$k = 5$ **OR/*OF*** $k = 15$ ✓ 10-5 $\quad$ 10+5	✓ $k = 5$ **OR/*OF*** ✓$k = 15$	(2)

CA $k = 10 \pm r$

$x = k$
vertical line

4.4	Coordinates of the point directly opposite N is C. *Koördinate van die punt regoor* N *is* C. $\frac{x+6}{2} = 10 \quad \frac{y-8}{2} = -5$ ✓ C(14; −2) ✓ ✓ Equation of the tangent at C: *Vergelyking van die raaklyn by* C: $y + 2 = -\frac{4}{3}(x - 14)$ $y = -\frac{4}{3}x + \frac{50}{3}$ ✓ $\therefore 0 < t < \frac{50}{3}$ ✓ $y = -\frac{4}{3}x + c$ Sub C(14; −2) $-2 = -\frac{4}{3}(14) + c$ ✓ $\frac{50}{3} = c$ → 16,67 6	✓ formula and sub / *formule en vervanging* ✓ for x-coordinate *vir x-koördinaat* ✓ for y-coordinate *vir y-koördinaat* ✓ substitution / vervanging ✓ for the answer / *vir die antwoord* ✓ for the value of t. *vir die waarde van t*	(6)
4.5	They will not touch. ✓ The new circle is the old circle shifted up by 11. ✓R 2 *Hulle sal nie raak nie.* *Die nuwe sirkel is die ou sirkel* 11 *eenhede opwaarts geskuif.*	✓ answer / *antwoord* ✓ any valid reason / *enige geldige antwoord*	(2)
			[22]

OR

M(10; −5) M′(10; 6)

r = 5 r′ = 5

$d_{MN'} = 11$ $r + r' = 10$

> 10

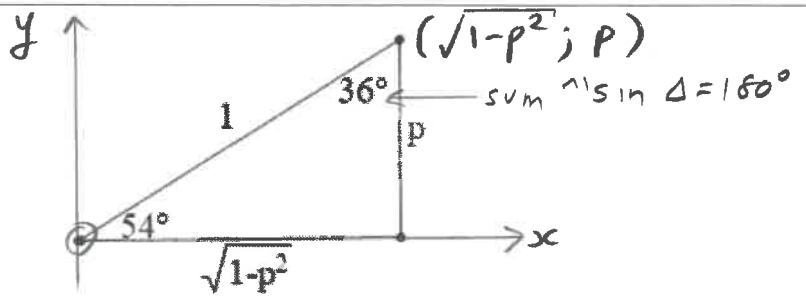
∴ circles will not touch

QUESTION 5/*VRAAG 5*

$\sin 54° = p$

$= \frac{p}{1} \quad \frac{y}{r}$

$x = \sqrt{1-p^2}$ Pythag



5.1.1	$\sin 594°$ $= \sin 234°$ $\sin(180°+54°)$ $= -\sin 54°$ ✓ $= -p$ ✓ (2)	✓ $-\sin 54°$ ✓ $-p$	(2)
5.1.2	$\cos 36^0$ $= p$ ✓✓ (2)	✓✓ answer / *antwoord*	(2)
5.1.3 PTO	$\cos 18°$ $\cos(90°-72°)$ $= \sin 72°$ (OR) $= \sin 2(36°)$ $\sqrt{\frac{p+1}{2}}$ $= 2\sin 36°\cos 36°$ $= 2\left(\frac{\sqrt{1-p^2}}{1}\right)\left(\frac{p}{1}\right)$ $= 2p\left(\sqrt{1-p^2}\right)$	✓ for / *vir* $\sin 72°$ ✓ for / *vir* $\sin 2(36°)$ ✓ for / *vir* $2\sin 36°\cos 36°$ ✓ for / *vir* $2p\left(\sqrt{1-p^2}\right)$	(4)
5.2	$\frac{\cos 140° - \sin(90°-\theta)}{\sin 410° + \cos(-\theta)}$ $= \frac{\cos(90°+50°) - \cos\theta}{\sin 50° + \cos\theta}$ $= \frac{-\sin 50° - \cos\theta}{\sin 50° + \cos\theta}$ $= \frac{-(\sin 50° + \cos\theta)}{(\sin 50° + \cos\theta)}$ $= -1$	✓ $-\sin 50°$ ✓ $\cos\theta$ ✓ $\sin 50°$ ✓ $\cos\theta$ ✓ for the common factor *vir die gemene faktor* ✓ for the answer *vir die antwoord*	(6)

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5.13. $\cos 2x = 2\cos^2 x - 1$

$x = 18^\circ$

$\cos 36^\circ = 2\cos^2 18^\circ - 1$ ✓

$p = 2\cos^2 18^\circ - 1$ ✓

(4)

$\frac{p+1}{2} = \cos^2 18^\circ$ ✓

$\sqrt{\frac{p+1}{2}} = \cos 18^\circ$ ✓ ± but reject -18° QI

5.2 $\frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 410^\circ + \cos(-\theta)}$

- $\cos 140^\circ = \cos(90^\circ + 50^\circ)$
  $= -\sin 50^\circ$
- $\sin 410^\circ = \sin 50^\circ$
- $\sin(90^\circ - \theta) = \cos\theta$
- $\cos(-\theta) = \cos\theta$

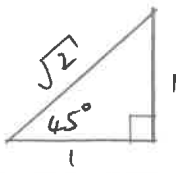
$\therefore \frac{-\sin 50^\circ - (\cos\theta)}{\sin 50^\circ + (\cos\theta)}$ ✓ ✓ ✓

$= \frac{-\sin 50^\circ - \cos\theta}{\sin 50^\circ + \cos\theta}$

$= \frac{-(\sin 50^\circ + \cos\theta)}{\sin 50^\circ + \cos\theta}$ ✓

(6)

$= -1$ ✓

5.3 PTO	$\cos(x+65°).\cos(x+20°) - \sin(x+245°).\sin(x+20°)$ $= \cos(x+65°).\cos(x+20°) + \sin(x+65°).\sin(x+20°)$ $= \cos[(x+65°)-(x+20°)]$ $= \cos 45°$ $= \frac{1}{\sqrt{2}}$ 4  	✓ reduction / *reduksie* ✓ compound angle *saamgestelde hoek* ✓ $\cos 45°$ ✓ answer / *antwoord*	(4)
5.4 PTO	$\cos^2 x - \sin^2 x = \frac{1}{2}$ $\cos 2x = \frac{1}{2}$ $2x = 60° + 360°.k$ or/*of* $2x = 300° + 360°.k$ $x = 30° + 180.k$ or/*of* $x = 150° + 180.k$ $k \in Z$ 4 **OR / OF** $2\cos^2 x - 2\sin^2 x = 1$ $2\cos^2 x - 2\sin^2 x = \sin^2 x + \cos^2 x$ $3\sin^2 x = \cos^2 x$ $\tan^2 x = \frac{1}{3}$ ∴ $\tan x = \pm\sqrt{\frac{1}{3}}$ ref ∠ = 30° tan ± in I : $x = 30° + k\,180°$; $k \in Z$ or II : $x = 150° + k\,180°$; $k \in Z$	✓ $\cos 2x = \frac{1}{2}$ ✓ for / *vir* $2x$ in both quadrants / *in beide kwadrante* ✓ $x = 30° + 180.k$ ✓ $x = 150° + 180.k$ **OR / OF** ✓ multiplying by 2 and using identity / *vermenigvuldig met 2 en gebruik van identiteit* ✓ $3\sin^2 x = \cos^2 x$ ✓ $\tan x = \frac{1}{\sqrt{3}}$ ✓ answer / *antwoord*	(4)
5.5.1	LHS $= \frac{\sin 2\theta . \tan\theta}{\cos 2\theta + 1}$ $= \frac{2\sin\theta\cos\theta . \frac{\sin\theta}{\cos\theta}}{2\cos^2\theta - 1 + 1}$ $= \frac{2\sin^2\theta}{2\cos^2\theta}$ $= \tan^2\theta$ 4	✓ $2\sin\theta\cos\theta$ ✓ $\frac{\sin\theta}{\cos\theta}$ ✓ $2\cos^2\theta - 1$ ✓ $\frac{2\sin^2\theta}{2\cos^2\theta}$	(4)
5.5.2 PTO	$\cos 2\theta + 1 = 0$ $\cos 2\theta = -1$ $2\theta = 180°$ $\theta = 90°$ 4	✓ $\cos 2\theta + 1 = 0$ ✓ $\cos 2\theta = -1$ ✓ $2\theta = 180°$ ✓ $\theta = 90°$	(4)
			[30]

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5.3. $\cos(x+65°)\cos(x+10°) - \sin(x+245°)\sin(x+20°)$

- $\sin(x+245°) = \sin(x+180°+65°)$
  $= \sin(180°+x+65°)$
  $= \sin(180°+(x+65°))$
  $= -\sin(x+65°)$

$\therefore \cos(x+65°)\cos(x+10°) - (-\sin(x+65°))\sin(x+10°)$
$= \cos(x+65°)\cos(x+10°) + \sin(x+65°)\sin(x+10°)$
etc

5.4. $\cos^2 x - \sin^2 x = \frac{1}{2}$

$\cos 2x = \frac{1}{2}$

$\cos A = \frac{1}{2}$ $\quad A = 2x$

ref $\angle = 60°$

cos + in

I: $A = 60° + k\cdot 360°$ or IV: $A = 300° + k\cdot 360°$

$2x = 60° + k\cdot 360°$ $\quad 2x = 300° + k\cdot 360°$

$x = 30° + k\cdot 180°; k \in \mathbb{Z}$ $\quad x = 150° + k\cdot 180°; k \in \mathbb{Z}$

OR

$\cos^2 x - \sin^2 x = \frac{1}{2}$

$1 - \sin^2 x - \sin^2 x = \frac{1}{2}$

$-2\sin^2 x = -\frac{1}{2}$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

ref $\angle = 30°$

Sin ± in

I : $x = 30° + k\,360°;\ k \in \mathbb{Z}$

II : $x = 150° + k\,360°;\ k \in \mathbb{Z}$

III : $x = 210° + k\,360°;\ k \in \mathbb{Z}$

or

IV : $x = 330° + k.360°;\ k \in \mathbb{Z}$

5.5.2. ID undefined when

NB : $0° \leq \theta \leq 180°$

- $\cos 2\theta + 1 = 0$ ✓

  $A = 2\theta$

  $\cos A = -1$ ✓

  $A = 180° + k.360°$

  $2\theta = 180° + k.360°$ ✓

  $\theta = 90° + k.180°$

  $\therefore \theta = 90°$ ✓

- $\tan\theta = UD$

  $\frac{\sin\theta}{\cos\theta} = UD$

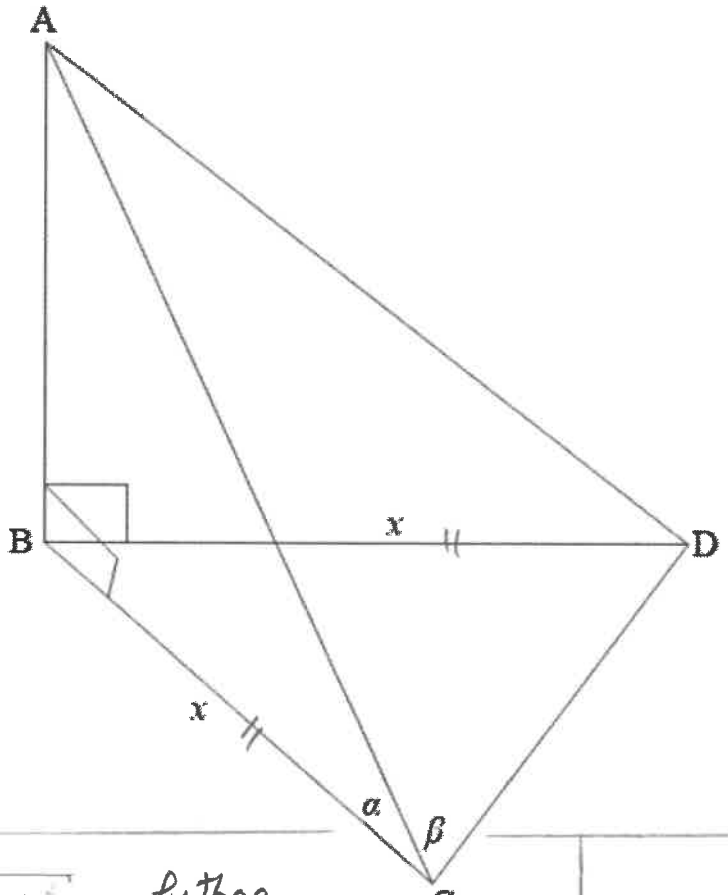
  $\cos\theta = 0$

  $\theta = 90° + k.180°$

  – done!

  $(k \in \mathbb{Z})$

QUESTION 6 / *VRAAG 6*



6.1	$AC = \sqrt{AB^2 + x^2}$ Pythag $AD = \sqrt{AB^2 + x^2}$ Pythag $\therefore AC = AD$ both $= \sqrt{AB^2 + x^2}$ ✓SR (1)	✓ S and/*en* R	(1)
6.2	$\cos\alpha = \frac{x}{AC}$ ✓ $AC = \frac{x}{\cos\alpha}$ ✓ (2)	✓ S ✓ answer / *antwoord*	(2)
6.3 PTO	$CD^2 = \left(\frac{x}{\cos\alpha}\right)^2 + \left(\frac{x}{\cos\alpha}\right)^2 - 2\left(\frac{x}{\cos\alpha}\right)\left(\frac{x}{\cos\alpha}\right)\cos(180° - 2\beta)$ $CD^2 = \frac{x^2}{\cos^2\alpha} + \frac{x^2}{\cos^2\alpha} + 2\left(\frac{x^2}{\cos^2\alpha}\right)\cos 2\beta$ $CD^2 = \frac{2x^2}{\cos^2\alpha}(1 + \cos 2\beta)$ $CD^2 = \frac{2x^2}{\cos^2\alpha}(1 + (1 - 2\sin^2\beta))$ $CD^2 = \frac{2x^2}{\cos^2\alpha} \times 2\cos^2\beta$ $CD^2 = \frac{4x^2\cos^2\beta}{\cos^2\alpha}$ $\therefore CD = \frac{2x\cos\beta}{\cos\alpha}$ **OR / *OF***	✓ cos rule / *cos-reël* ✓ simplification *vereenvoudiging* ✓ double angles expansion *dubbelhoeke uitbreiding* ✓ simplification *vereenvoudiging* **OR / *OF***	

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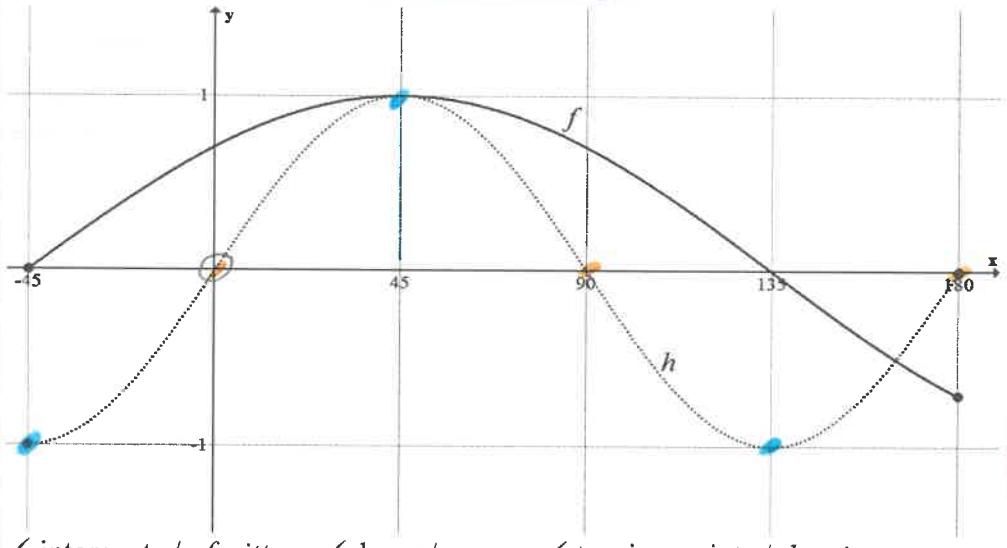
	$A\hat{D}C = \beta$ $C\hat{A}D = 180^0 - 2\beta$ $\frac{CD}{\sin(180^0 - 2\beta)} = \frac{AC}{\sin\beta}$ $CD = \frac{AC \sin 2\beta}{\sin\beta}$ $CD = \frac{x.\,2\sin\beta\cos\beta}{\cos\alpha \sin\beta}$ $CD = \frac{2x\cos\beta}{\cos\alpha}$	✓ for /*vir* $A\hat{D}C = \beta$ and/*en* $C\hat{A}D = 180^0 - 2\beta$ ✓ use of sin rule *gebruik van sinus-reël* ✓ substitution of AC *vervanging van* AC ✓ simplification / *vereenvoudiging*	(4)
6.4	$CD = \frac{2x\cos\beta}{\cos\alpha}$ $CD = \frac{2(25)\cos 65{,}62^\circ}{\cos 30^\circ}$ $CD = 23{,}83$ cm	✓ substitution / *vereenvoudiging* ✓ answer / *antwoord*	(2)
			[9]

QUESTION 7 / *VRAAG 7*

7.1	$f(180°) = -0{,}71$ ∴ Range: *Terrein: Waardeversameling*: $-0{,}71 \leq y \leq 1$ **OR** $y \in [-0{,}71;\ 1]$	✓ $f(180°)$ ✓ answer / *antwoord*	(2)
7.2	 ✓ intercepts / *afsnitte* ✓ shape / *vorm* ✓ turning points / *draaipunte*		(3)
7.3	Period / *Periode* = 180°	✓ answer / *antwoord*	(1)
7.4	$-45° < x < 45°$	✓✓ answer / *antwoord*	(2)
7.5	$x = -45°$ or / *of* $x = 135°$	✓ $x = -45°$ ✓ $x = 135°$	(2)
7.6	$g(x) = \cos(x + 15°)$	✓ answer / *antwoord*	(1)
			[11]

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6.3. ✓ $A\hat{D}C = \beta$ ∠s opp = sides (6.1.)

✓ $C\hat{A}D = 180° - 2\beta$ sum ∠s in $\Delta = 180°$

$$\frac{CD}{\sin(180° - 2\beta)} = \frac{AC}{\sin\beta}$$ ✓

$$\frac{CD}{\sin 2\beta} = \frac{\frac{x}{\cos\alpha}}{\sin\beta}$$ ✓ AC

$\div \sin\beta$ / $\times \frac{1}{\sin\beta}$

$$= \frac{x}{\cos\alpha} \times \frac{1}{\sin\beta}$$

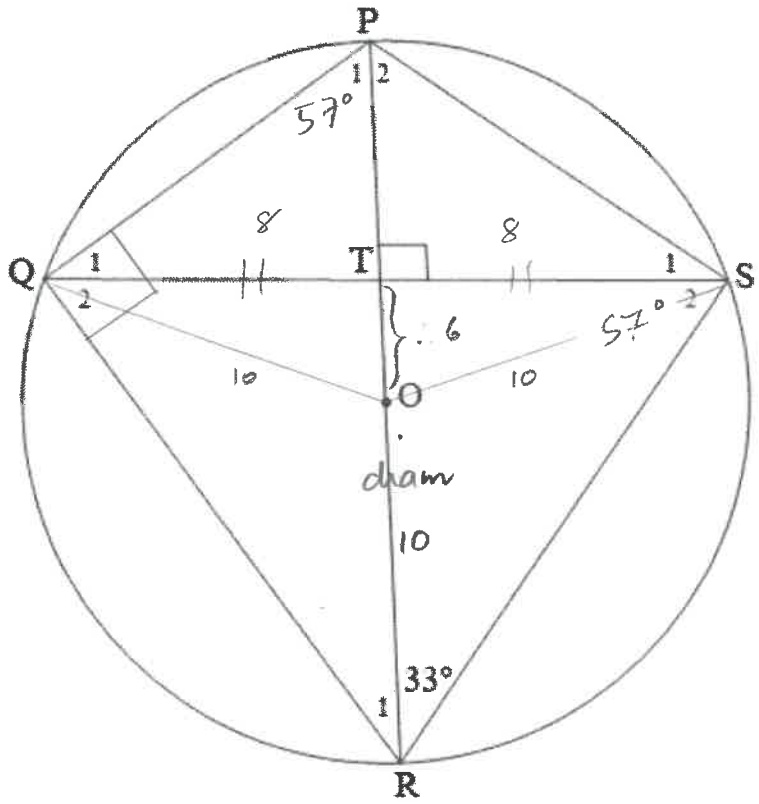
$$CD = \frac{x \cdot \sin 2\beta}{\cos\alpha \sin\beta}$$

$$= \frac{x \cdot 2\sin\beta\cos\beta}{\cos\alpha \sin\beta}$$ ✓

$$= \frac{2x\cos\beta}{\sin\alpha}$$ D

(4)

QUESTION 8 / *VRAAG 8*

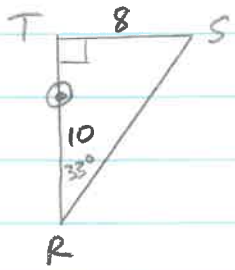


8.1.1	$\hat{Q}_1 = 33°$ (∠s in the same segment) / (*∠e in dieselfde segment*)	✓ S ✓ R	
	$\hat{P}_1 = 57°$ (∠s of a triangle) / (*∠e van 'n driehoek*)	✓S and/*en* R	(3)
8.1.2	$P\hat{Q}R = 90°$ (∠ subtended by the diameter) / (*∠ onderspan deur die middellyn*)	✓ S and/*en* R	
	$\hat{Q}_2 = 57°$ (complementary ∠s / ∠s of a triangle) (*komplementêre ∠e / ∠e van 'n driehoek*)	✓ S and/*en* R	(2)
8.2	$QT = TS = 8\ cm$ (line from centre perp to chord) (*lyn vanaf middelpunt loodreg op koord*)	✓ S and/*en* R	
	$OQ = OS = 10\ cm$ (radii) / (*radiusse*)	✓ S and/*en* R	
	$OQ^2 = TO^2 + QT^2$ (Pythagoras) / (*Pythagoras*)	✓ S and/*en* R	
	$10^2 = OT^2 + 8^2$ $TO = \sqrt{100 - 64} = 6$ cm	✓ answer / *antwoord*	(4)
			[9]

TO → 4,80
→ 2,32
→ 2,61

pg 11.5.

8.2 $TS = 8$ line from centre ⊙ ⊥ to chord



$\frac{8}{TR} = \tan 33°$

$8 = TR \tan 33°$

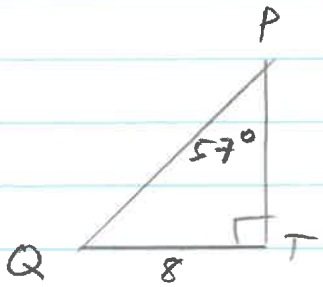
$\frac{8}{\tan 33°} = TR$

$12,31\ldots = $

$\therefore TO = 12,31\ldots - 10$

$= 2,32$

OR



$QT = 8$ line from centre ⊙ ⊥ to chord

$\frac{8}{PT} = \tan 57°$

$8 = PT \tan 57°$

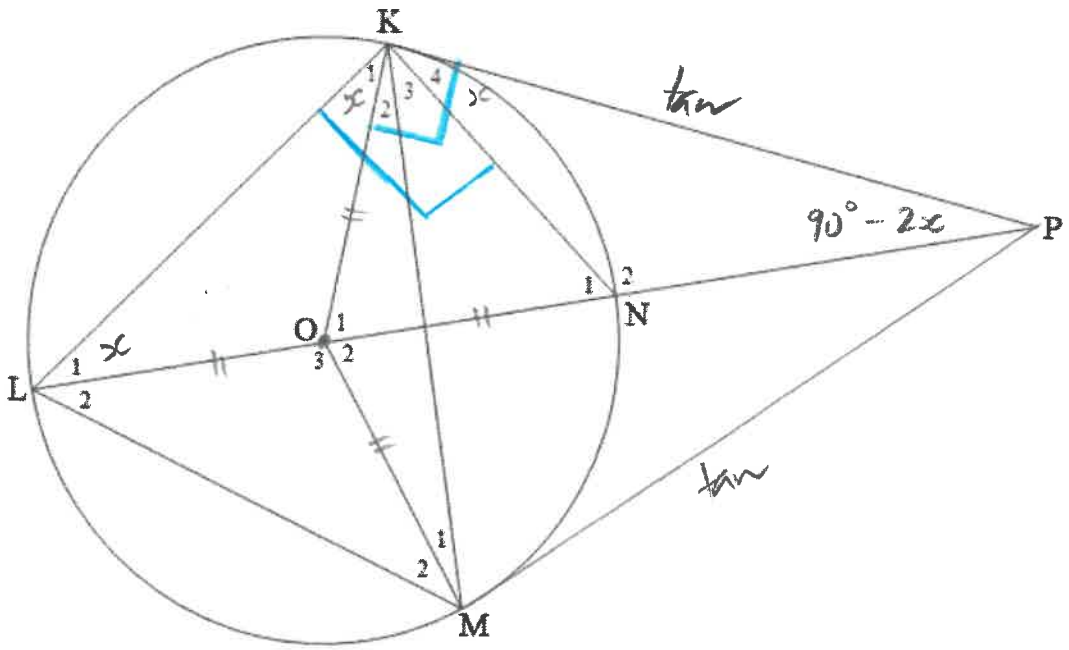
$\frac{8}{\tan 57°} = PT$

$5,19\ldots = PT$

$\therefore TO = 10 - 5,19\ldots$

$= 4,80$

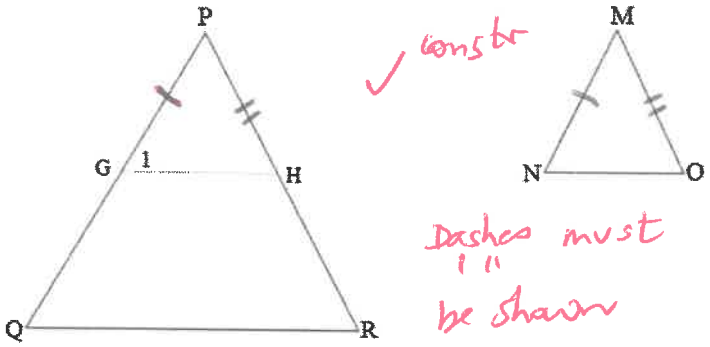
QUESTION 9 / *VRAAG 9*



9.1	$L\hat{K}N = 90°$ ✓SR ∧ in semi ⊙ = 90° $O\hat{K}P = 90°$ ✓SR tan ⊥ rad (2)	✓ answer / *antwoord* ✓ answer / *antwoord*	(2)
9.2.1	$\hat{L}_1 = x$ (tan – chord theorem) / (*raaklyn-koord stelling*) ✓S ✓R (2)	✓ S ✓ R	(2)
9.2.2	$\hat{K}_1 = x$ (∠s opp. = sides, radii) (2)	✓ S ✓ R	(2)
9.2.3	In ΔLKP; $L\hat{K}P = 90° + x$ and/*en* $\hat{L}_1 = x$ $\therefore \hat{P} = 90° - 2x$ (∠s of a triangle) (*∠e van 'n driehoek*) ✓✓ (2) **OR / *OF*** $\hat{N}_2 = 90^0 + x$ (*ext ∠ of Δ*) / (*buite ∠ van Δ*) $\therefore \hat{P} = 90° - 2x$ (∠s of a triangle) (*∠e van 'n driehoek*)	✓ S ✓ R **OR / *OF*** ✓ S/R ✓ S/R	(2)
9.3	$O\hat{M}P = 90°$ (tan⊥ rad) / (*raaklyn ⊥ radius*) ✓SR $O\hat{K}P = 90°$ (proven) / (*bewys*) ✓SR ∴ KOMP is a c.q. (conv opp ∠s cyclic quad = 180°) ✓R (3)	✓ S & R ✓ S ✓ R	(3)
			[11]

QUESTION 10 / *VRAAG 10*

10.1



✓ constr

Dashes must be shown

On PQ, mark PG = MN and on PR, mark off PH = MO
Join GH
In ΔPGH and ΔMNO
(1) PG = MN (construction)
(2) $\hat{P} = \hat{M}$ (given)
(3) PH = MO (construction)
$\therefore \Delta PGH \equiv \Delta MNO$ (SAS) ✓ SR
$\therefore \hat{G}_1 = \hat{N}$ $\Delta PGH \equiv \Delta MNO$
But $\hat{Q} = \hat{N}$ given
$\therefore \hat{G}_1 = \hat{Q}$ both = $\hat{N}$
$\therefore GH \parallel QR$ (corresponding angles =) ✓ SR
$\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (line ∥ to 1 side of Δ) ✓ SR
But PG = MN and PH = MO ✓ S constr
$\therefore \frac{MN}{PQ} = \frac{MO}{PR}$

✓ construction
✓ congruency proof
✓ S ✓ R
✓ S and/*en* R
✓ S

(6)

Afrikaans

Op PQ, *merk af* PG = MN *en op* PR, *merk af* PH = MO
Verbind GH
In ΔPGH *en* ΔMNO
(1) PG = MN (*konstruksie*)
(2) $\hat{P} = \hat{M}$ (*gegee*)
(3) PH = MO (*konstruksie*)
$\therefore \Delta PGH \equiv \Delta MNO$ (SHS)
$\therefore \hat{G}_1 = \hat{N}$ (*Kongruensie*)
Maar, $\hat{Q} = \hat{N}$ (*gegee*)
$\therefore \hat{G}_1 = \hat{Q}$
$\therefore GH \parallel QR$ (*ooreenkomstige hoeke gevorm* =)
$\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (*eweredigheid*; $GH \parallel QR$)
Maar, PG = MN *en* PH = MO
$\therefore \frac{MN}{PQ} = \frac{MO}{PR}$

✓ *konstruksie*
✓ *kongruensie bewys*
✓ S ✓ R
✓ S and/*en* R
✓ S

(6)

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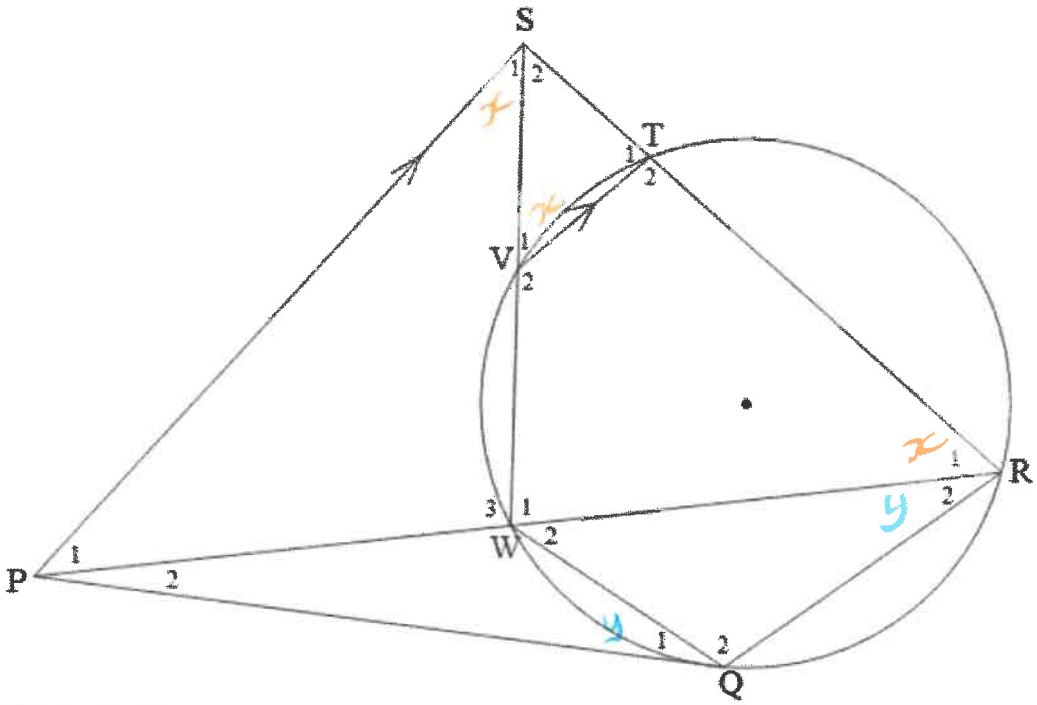

10.2.1	$\hat{S}_1 = \hat{V}_1$ (alt ∠s; PS||VT) / (*verw. ∠e ; PS || VT*) $\hat{V}_1 = \hat{R}_1$ (ext.∠ of cq RTVW) / (*buite ∠ van kv* RTVW) $\therefore \hat{S}_1 = \hat{R}_1$ both = $\hat{V}_1$ (3)	✓ S and/*en* R ✓ S ✓ R	(3)
10.2.2	In ΔPWS and/*en* ΔPSR (1) $\hat{P}_1$ is common / *is gemeen* (2) $\hat{S}_1 = \hat{R}_1$ (proven) / (*bewys*) (3) 3rd angle of a triangle / (3de *hoek van driehoek*) ΔPWS ||| ΔPSR (AAA) / (HHH) / (∠∠∠) (3)	✓ S ✓ R ✓ S and/*en* R	(3)
10.2.3	In ΔPQW and/*en* ΔPRQ (1) $\hat{Q}_1 = \hat{R}_2$ (tan – chord theorem) /(*raaklyn – koord stelling*) (2) $\hat{P}_2$ is common / *is gemeen* (3) 3rd angle of a triangle / 3de *hoek van driehoek* ΔPQW ||| ΔPRQ (AAA) $\therefore \frac{PQ}{PR} = \frac{PW}{PQ}$ ΔPQW ||| ΔPRQ (5) $\therefore PQ^2 = PW.PR$	✓ S and/*en* R ✓ S ✓ S and/*en* R ✓ S ✓ S	(5)
10.2.4	From / *Vanaf* 10.2.1 $\frac{PW}{PS} = \frac{PS}{SR}$ (ΔPWS ||| ΔPSR) $\therefore PS^2 = PW.PR$ (3) From / *Vanaf* 10.2.2 $PQ^2 = PW.PR$ $\therefore PQ^2 = PS^2$ both = PW.PR $\therefore PQ = PS$	✓ S ✓ S ✓ conclusion	(3)
			[20]

TOTAL / *TOTAAL*: 150

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